Analysis of Sheet Variations – Insights of Two-Dimensional Variations

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ABSTRACT

The variations of sheet properties are often expressed as two-dimensional (2D) functions in both machine direction (MD) and cross-machine direction (CD). The traditional variance partition analysis (VPA) focuses on dividing the total variance into three components, MD, CD and Residual. Even though VPA has been widely used in the industry for many decades, it gives no clear insights about the characteristics of each component. This paper is intended to illustrate how to use power spectral analysis (PSA) and principal component analysis (PCA) to extract detectable patterns from a 2D sheet measurement. The extracted patterns can be associated to potential root causes in sheet-making processes or used to improve the uniformity of sheet variations.

INTRODUCTION

In a sheet-making process, the measurements of sheet variations are typically obtained from sensors that are traversing back and forth between the web edges while the sheet is made. The measurement taken along the trace from one edge to the other is called a “scan measurement”. The collection of multiple consecutive scans is the raw data to represent a two-dimensional sheet variation. Most commonly, this two-dimensional sheet variation is analyzed with a variance partition analysis (VPA) [1-6]. A traditional variance partition analysis designates the average of each scan as the MD component and the average of each databox (or slice in machine direction) as the CD component. The remaining variation is called “Residual”. The variance associated with each component is calculated with various algorithms. The separated variances often give papermakers a general indication whether the sheet is within its product specifications or not. VPA does not give papermakers any further details on the characteristics of each separated component. In reality, a sheet with truly random variations and a sheet with discernible patterns could have the exact same VPA variances.

In order to know whether a sheet is actually uniform or not, other analysis techniques will have to be used. One of the most commonly used techniques is the power spectral analysis (PSA) [10]. Power spectral analysis is based on the Fourier transformation of a variation function. There are many commercial software tools available for conducting the spectral analysis. Most of these tools are capable to apply Fast Fourier Transformation (FFT) on data such as a scan measurement. Power spectral analysis is particularly useful to analyze single-dimensional data even though it can also be extended to handle multiple dimensions. This paper will illustrate how PSA can be applied to extract the dominant variations in either MD or CD directions. The application of PSA on the 2D Residual component will also be discussed.

The other widely used method is the principal component analysis (PCA) [11]. Principal component analysis does not analyze each individual dimension separately. Instead, PCA analyzes a variation in both directions simultaneously. This technique is particularly useful for extracting the detectable patterns that are embedded in the Residual component. The application of PCA to detect and separate variation patterns will be illustrated in this paper as well.

These two analysis methods have different strengths and they complement each other. The goal of this paper is to demonstrate how these two methods can be utilized together to analyze a 2D sheet variation. Measurements from a real paper machine are used in the examples showing the application of these techniques.

POWER SPECTRAL ANALYSIS (PSA)

Power spectral analysis is an application of Fourier transformation on a variation function. Fourier transformation is a well-known method to analyze the frequency contents of a given function. In practice, Fourier transfer applied on an array of data could be accomplished by using a fast Fourier transformation (FFT) algorithm or more generally a discrete Fourier transformation (DFT) algorithm. The output of FFT or DFT is an array of complex numbers. The magnitudes of these complex numbers represent the spectral contents of the given variation at different frequencies.

Figure 1(a) shows an example of a variation function and Figure 1(b) is the power spectrum of the variation function in Figure 1(a).

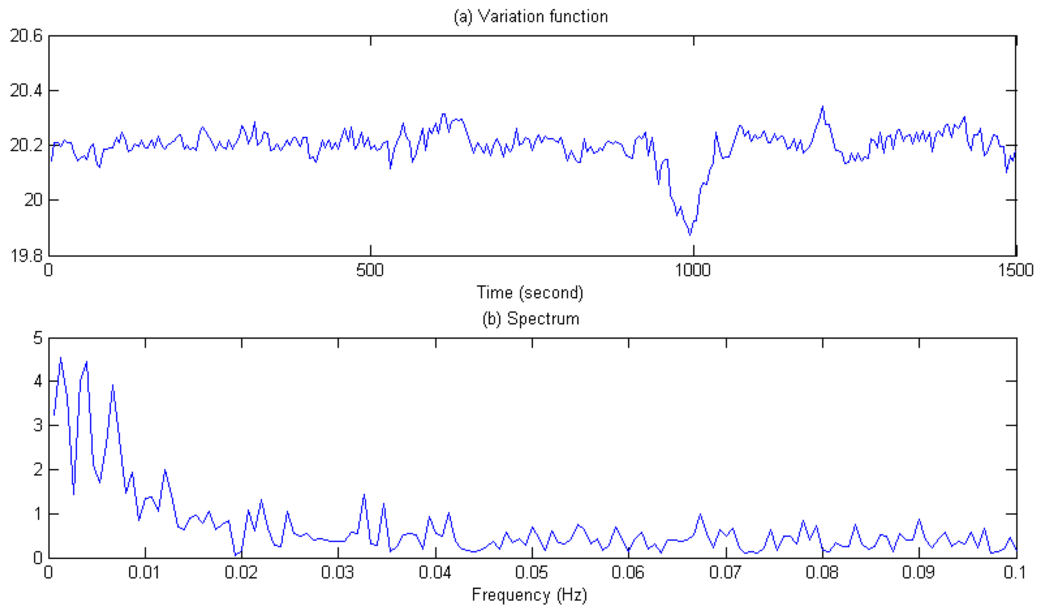

Figure 1: Example of a variation function and its power spectrum.

For a two-dimensional sheet variation, PSA could be applied in either MD or CD directions separately. The frequency content in the MD direction is often linked to process issues like furnish instability, pulsations, vibrations and flow or control oscillations. On the other hand, the frequency content in the CD direction is usually associated with non-uniformity across the sheet width, which can be caused by headbox unbalance, flow distribution variations, drainage unevenness, dryer can temperature gradients or the non-uniform spatial characteristics of various CD actuators. Typical measurements of sheet variations are obtained from online scanning sensors, which mean that some variation contents overlap between MD and CD power spectra.

A typical 2D sheet measurement $u(x,y)$ can be seen as a collection of m scans of measurement across the web width where each scan is measured with n databoxes, or slice positions, in CD direction.

$$u(x,y)=\begin{bmatrix} u_{11} & u_{12} & \cdot & \cdot & u_{1j} & \cdot & \cdot & u_{1n} \\ u_{21} & u_{22} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ u_{i1} & \cdot & \cdot & \cdot & u_{ij} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ u_{m1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & u_{mn} \end{bmatrix} \quad (1)$$

where i = 1 to m is the index of each individual profile and j = 1 to n is the index of each databox in a profile.

Using the $m\times n$ matrix $u(x,y)$, the following averages can be derived [3, 4].

The average CD profile is:

$$\bar{u}_y = \sum_{i=1}^{m}\frac{u_{ij}}{m} \quad (2)$$

The average MD trend is:

$$\bar{u}_x = \sum_{j=1}^{n}\frac{u_{ij}}{n} \quad (3)$$

The Residual is:

$$r(x,y)=[r_{ij}]; \quad r_{ij}=u_{ij}-\bar{u}_x(i)-\bar{u}_y(j)+\bar{u} \quad (4)$$

where the overall average is defined as :

$$\bar{u}=\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{u_{ij}}{nm} \quad (5)$$

PSA could be applied to each individual scan $u(i,y)$, each individual databox $u(x,j)$, the averaged CD profile $\bar{u}_y$, or the averaged MD trend $\bar{u}_x$. The spectrum of individual scans reveals the short-term variability of the CD profile in time, and the spectrum of individual databoxes highlights the local stability at each CD location. The spectrum of the averaged CD profile captures the persistent CD variation patterns, and the spectrum of the averaged MD trend is a good indication of the overall MD variation sources.

Figure 2 shows an example of 2D variation which is measured from an online caliper sensor. This 2D variation can be sliced in either CD or MD directions as indicated in Figure 3(a) and Figure 4(a) respectively. The averaged CD profile and averaged MD trend are shown in Figure 3(b) and Figure 4(b) respectively.

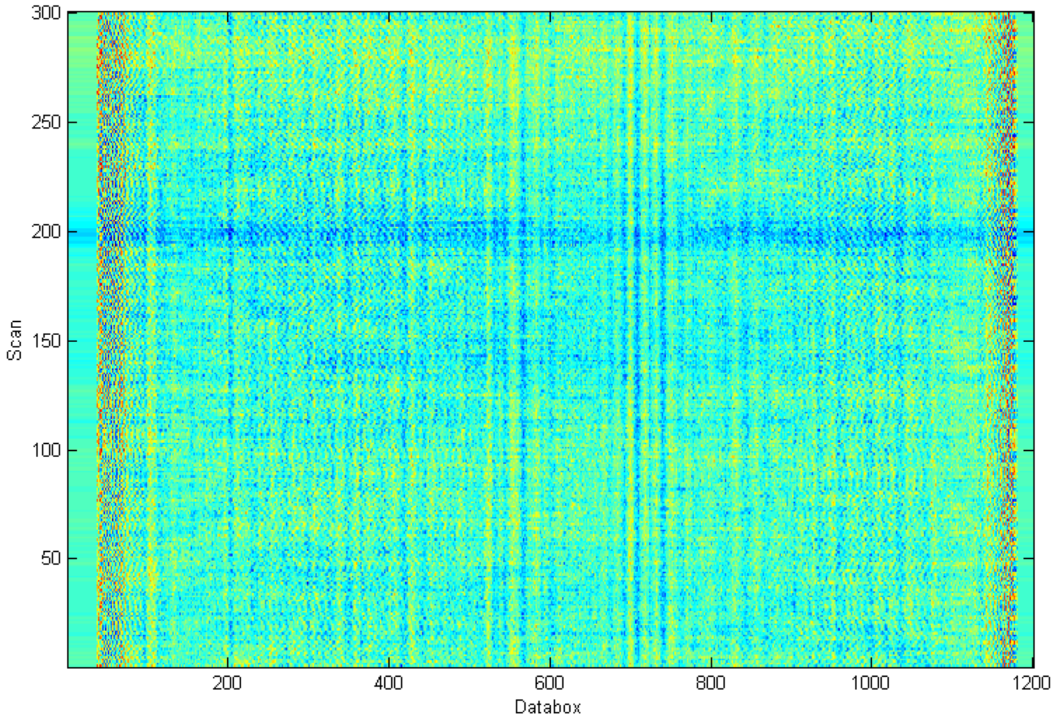

Figure 2 Example of a 2D sheet variation

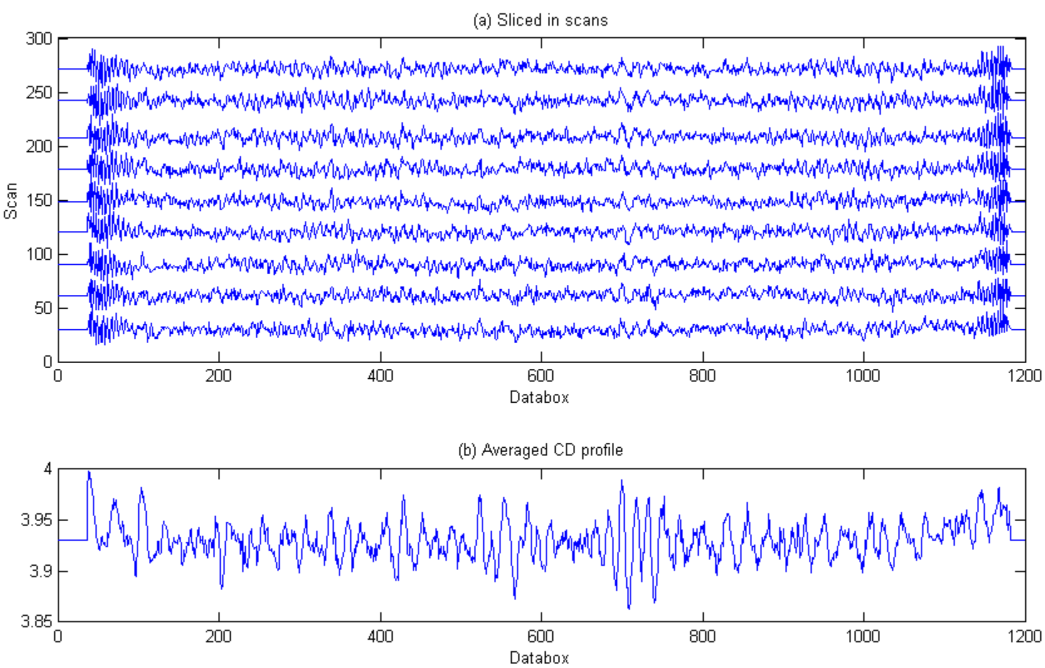

Figure 3 Variation from Figure 2 sliced in MD direction (scans)

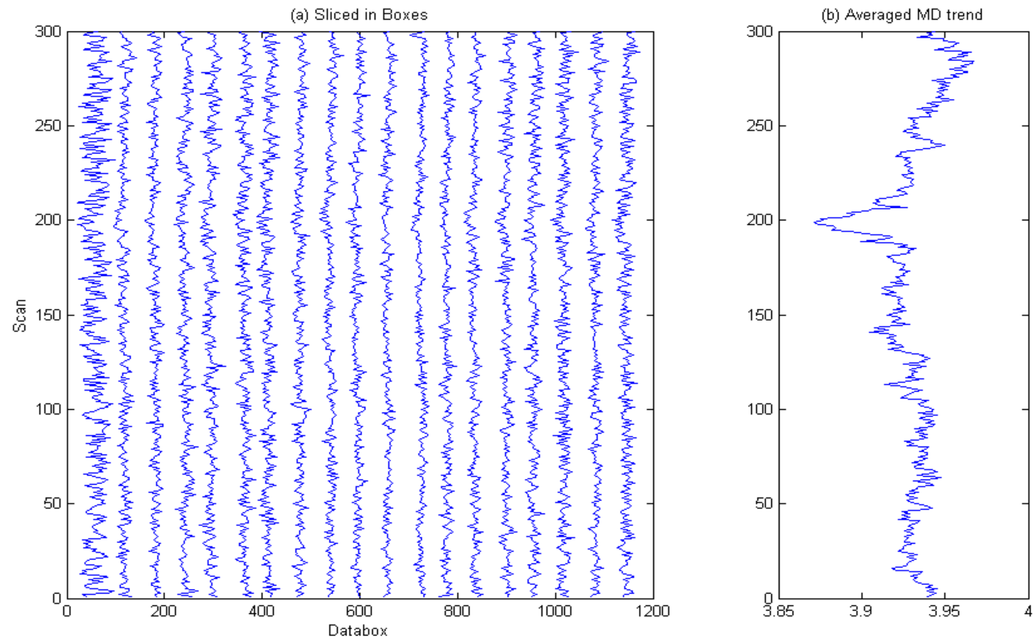

Figure 4 Variation from Figure 2 sliced in MD direction (databoxes)

The spectra of individual scans and the average of individual spectra are shown in Figure 5(a) and (b) respectively. The spectra of individual databoxes and the average of individual spectra are shown in Figure 6(a) and (b) respectively. The spectrum of the averaged CD profile is also displayed (red line) in Figure 5(b). Similarly, the spectrum of the averaged MD trend is displayed (red line) in Figure 6(b) as well.

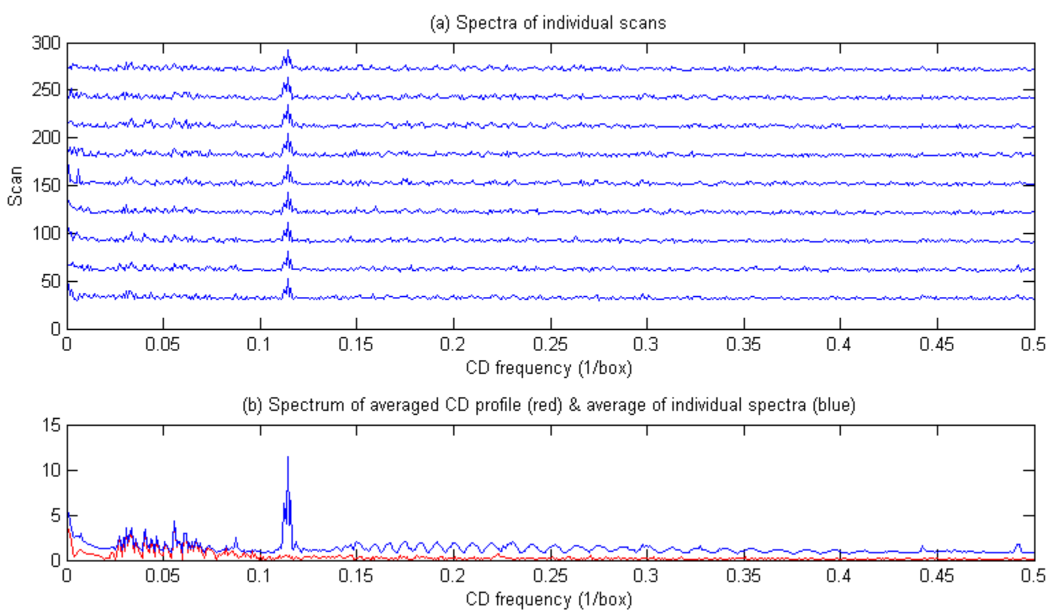

Figure 5 Spectra of individual scans, the averaged CD profile, and the average of individual spectra

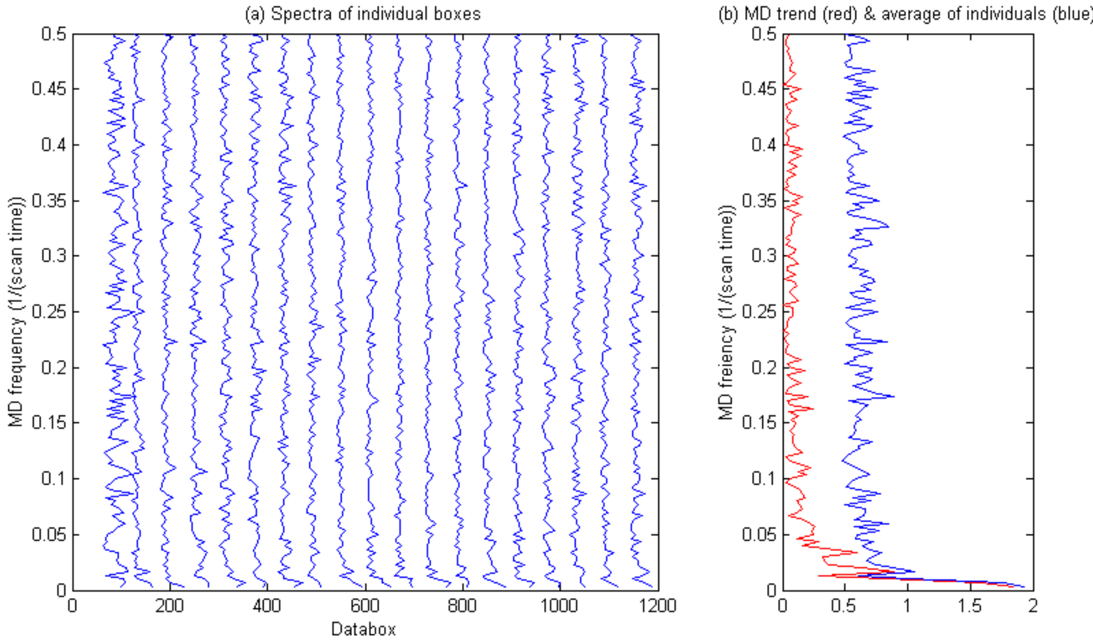

Figure 6 Spectra of individual databoxes, the averaged MD trend, and the average of individual spectra

For CD analysis, it is important to note that the spectrum of the averaged CD profile (red line in Figure 5(b)) is significantly different from the spectrum averaged from the spectra of individual scans (blue line in Figure 5(b)). This difference generally separates the persistent (static) variability from the short-term (dynamic) variations in the CD profiles. With VPA analysis, only the persistent CD variability is accounted for in its CD variance. In fact for some applications as shown here, the short-term CD variability, which is left in the Residual, could be a significant portion of the overall CD variability. PSA technique is able to extract the short-term CD variations from the Residual.

As another key note, the short-term CD variability left in the Residual may actually contain some short-term MD variations. By using the scan speed (approximately 44 databoxes/second for this example), the spatial spectrum of the Residual (calculated directly from the Residual or approximated by the difference between the red and blue lines in Figure 5(b)) can be seen as a temporal spectrum as indicated in Figure 7. The peak spectral content near 5Hz may actually be a short-term MD variation, which could be further confirmed with the spectrum of a single-point measurement. Further discussion will be illustrated later.

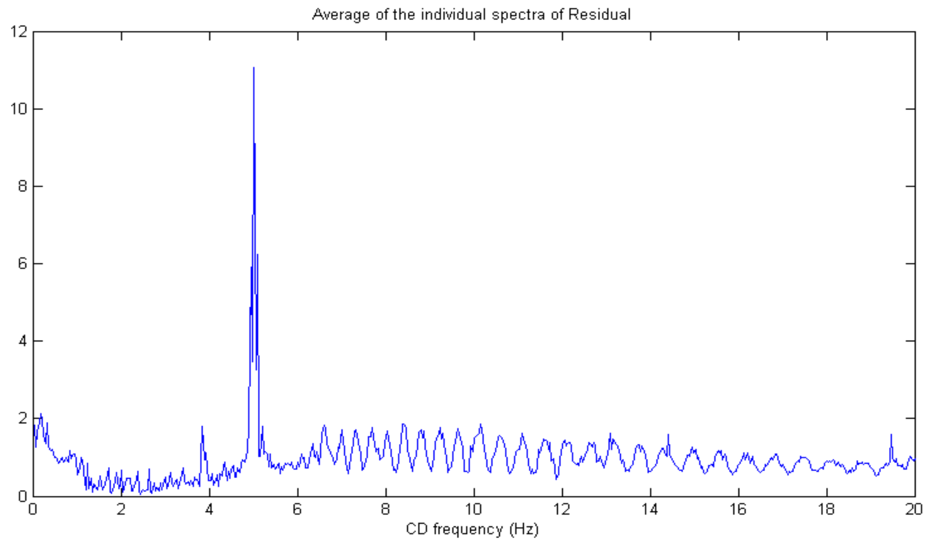

Figure 7 Equivalent temporal spectrum of the spatial spectrum of the Residual

For MD analysis, there are limitations on what information the scan data can reveal due to the fact that the scan data are often obtained from a zigzag scan path. The measurements at those databoxes that are not near the center of the sheet are obtained with uneven sampling period, which is alternating every other scan. MD spectral analysis without considering such effect could introduce distortion in the spectral content. To mitigate this issue, MD spectral analysis of individual databoxes could be performed on the odd and even scan data separately. The equivalent sampling period is doubled, but the calculated spectra are more precise.

For MD spectral analysis, the variations faster than two times the scan period can not be reliably extracted from the scanned data. The single-point faster-sampling measurement is better suited for analyzing fast MD variations. Depending on which sampling rate is used, the different range of spectral contents can be extracted. Figure 8 illustrates three measurements that are obtained with 200Hz sampling rate at a single point near one edge of the sheet. The spectral contents up to 100Hz are detectable with these measurements. The strong oscillation of the caliper measurement in the third plot window of Figure 8 indicates that the caliper has a strong variation at 5.1-5.2 Hz as shown in the bottom plot window. This frequency closely matches with the peak frequency in Figure 7. It means that the peak variation in short-term CD spectrum (in Figure 7) is indeed a fast MD variation instead. In fact if there is no single-point measurement available for comparison, the short-term MD variations could very well be treated as part of the short-term CD variations as you notice in this example.

Figure 8 Spectral analysis display

Detect Dominant Spectral Contents

Theoretically, a completely random variation should have a uniform spectrum for all frequencies. In reality, it is unlikely to see a completely uniform spectrum. If the spectrum is non-uniform, there are a few significantly larger magnitudes at certain frequencies. These frequencies are considered the "dominant frequencies", and their corresponding spectral components are the "dominant spectrum" in the given variation. A key step of applying spectral analysis is to identify the dominant spectral contents. The dominant spectral contents can be visually determined with a spectral plot. They can also be identified with various techniques. For the example in Figure 8, the dominant frequencies of the caliper spectrum (the bottom plot window) are marked with the red dots in the spectral plot.

Link Dominant Frequency with Potential Root Causes

With the dominant spectral contents identified, their corresponding frequencies can be associated with potential root causes. For CD spectra, the dominant spatial frequencies are often inverted to become the dominant wavelengths in CD direction. The dominant spatial wavelengths are linked to actuator distributions, alignments, spatial response widths or other physical dimensions in CD direction.

For MD spectra, the dominant frequencies can be linked to the diameters of rolls, the length of felts with a machine speed measurement, or associated with the vibration frequencies of rotating devices in a paper machine. The example in Figure 8 shows that the dominant frequencies of 5.1-5.2 Hz in caliper measurement are closely matched with the diameter of the top calender roll. In fact, the faulty bearing of this roll causes the vibration, which induces 5.1-5.2 Hz oscillations in the actual caliper variations.

Reconstruct Dominant Variations

The inverse FFT can be used to reconstruct the dominant variation from the dominant spectral components. Overlaying the reconstructed dominant variation on the original variation will allow users to see the contribution of the dominant variation in the original variation. In Figure 8, the variation reconstructed with the marked dominant frequencies at 5.1-5.2 Hz is shown by the red line overlaying with the original measurement in the third plot window of Figure 8. If the reconstructed variation is subtracted from the original variation, the difference is a good indication of what variation might be left behind after the dominant variation is eliminated. To go a step further, taking a ratio between the remaining and the original variations, the potential variability reduction can be estimated if the dominant variation is removed. The calculation shown at the bottom right hand corner of Figure 8 is an example of the projected reduction when the vibration of the top calender roll is eliminated.

PRINCIPAL COMPONENT ANALYSIS (PCA)

While power spectral analysis is an effective method to analyze a sheet variation, it is typically used for the single-dimensional slices of a 2D variation. To analyze the variation in both directions simultaneously, you would need to use different methods. Principal component analysis is one of the methods that can be effectively used for this purpose.

Given a two-dimensional variation $u(x,y)$ expressed in equation (1), the averaged CD profile and the averaged MD trend can be easily separated with equations (2) and (3) respectively. The Residual $r(x,y)$ derived in equation (4) will be ready for PCA to perform the further analysis.

According to the principal component analysis, the Residual matrix $r(x,y)$ can be expressed as a linear combination of two sets of vectors $g_k(y)$ and $a_k(x)$ as:

$$r(x,y) = \sum_k c_k a_k(x) g_k^T(y) \qquad (6)$$

where c_k is the singular value of the covariance matrix of $r(x,y)$

$g_k(y)$ and $a_k(x)$ are normalized orthogonal vectors corresponding to the singular value c_k

If each $a_k(x)g_k^T(y)$ is treated as a 2D basis matrix $\varphi_k(x,y)$, then the 2D Residual variation $r(x,y)$ can be seen as the linear combination of multiple 2D basis matrices [7]:

$$r(x,y) = \sum_k c_k \varphi_k(x,y) \qquad (7)$$

If all components are sorted by the magnitude of c_k in the descending order, then the first few terms in (7) will be the dominant components which contribute the majority of variance to the Residual $r(x,y)$. It is important to note that the dominant components also effectively capture the most significant variation patterns in the Residual $r(x,y)$.

By using the same example from Figure 2, Figure 9 shows the result of the principal component analysis of the Residual $r(x,y)$. Figure 9(a) is the contour plot of the residual $r(x,y)$, Figure 9(b) and (c) illustrate the first nineteen vectors of $a_k(x)$ and $g_k(y)$ respectively and Figure 9(d) is the plot of the first nineteen singular values of principal components. In Figure 9(c), the leading CD principal components clearly capture the dominant variation patterns. These principal components are equivalent to the dominant variations reconstructed from the spectral analysis and they can be used to construct 2D variation patterns in the original 2D variation.

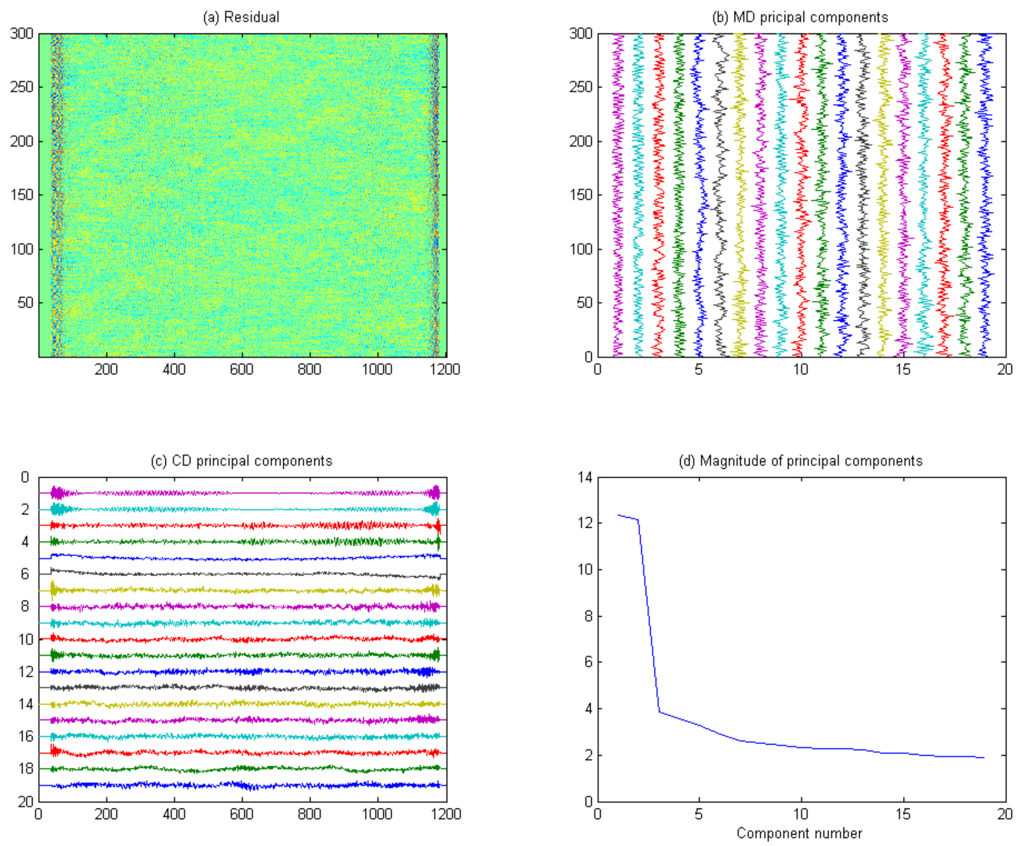

Figure 9 PCA decomposition of Residual variation $r(x,y)$

Construct Variation Patterns

When examining a Residual variation closely, quite often you may see a few variation patterns. Each variation pattern can be constructed from the 2D basis matrices as:

$$r(x,y) = \sum_{group1} c_k \varphi_k(x,y) + \sum_{group2} c_v \varphi_v(x,y) + \sum_{group3} c_w \varphi_w(x,y) + ... \qquad (8)$$

Figure 10 shows an example of constructing multiple 2D variation patterns by using the principal components and the magnitudes in Figure 9. For the current example, the Residual in Figure 10(a) consists of at least two detectable variation patterns. These two patterns are separately constructed and displayed in Figure 10(b) and (c). The remaining variation is shown in Figure 10(d).

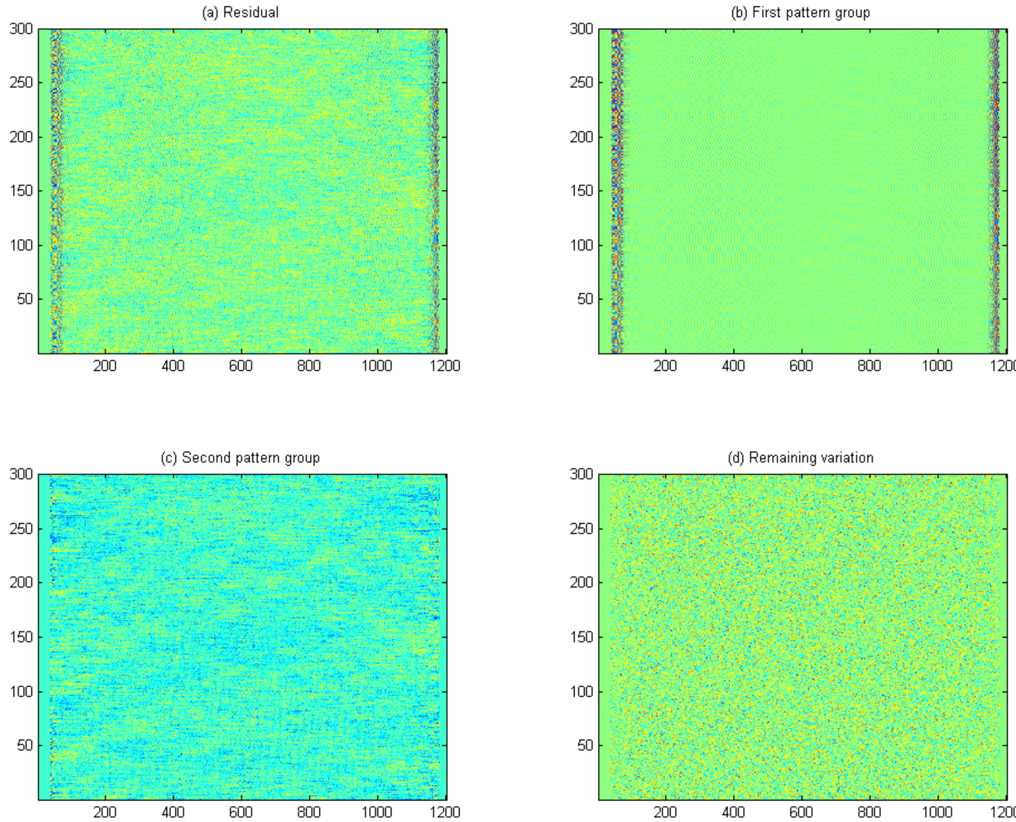

Figure 10 Construction of multiple groups of variation patterns

Link Variation Patterns to Potential Root Causes

The separated 2D variation patterns allow users to find out the potential root causes more easily. As an example, the variation pattern in Figure 10(b) is actually linked to the vibration of the faulty bearing in the machine as it was explained earlier. The second variation pattern in Figure 10(c) is possibly associated with the dynamics of a caliper CD control on the machine. The remaining variation in Figure 10(d) is closer to the truly random measurement that may not be linked to any specific root cause.

Evaluate Uniformity

The local variability indices [8, 9] can be applied to check the uniformity of all extracted variation patterns and the remaining variation. The local variability indices and variances of each extracted variation patterns could be used as additional metrics to the VPA analysis. By using the same example in Figure 2, Table 1 summarizes 2σ's, local variability indices (maximum local variability), and the ranges of dominant frequencies of the separated terms. The first 2D variation pattern as shown in Figure 10(b) has the highest local variability index of 1.95 and the remaining variation in Figure 10(d) has the lowest local variability at 1.09 which is very uniform. Also, the total variances of these two variation patterns are larger than those of MD and CD components. By removing the root causes for these two variation patterns, the sheet variability will be significantly improved.

As a final step of this analysis, the ratio between the remaining variation, and the original Residual, is a good indication of what reduction could be achieved if those major variation patterns were eliminated. For the example in Table 1, the Residual of VPA analysis potentially can be reduced by 59% if both variation patterns in Figure 10 are eliminated. This information is very useful to determine the performance impacts of different root causes.

	Total	MD	CD	Residual	Pattern 1	Pattern 2	Remaining
2σ	0.0983	0.0302	0.0374	0.0857	0.0604	0.0497	0.0351
Local variability index	1.51	1.36	1.51	1.61	1.95	1.29	1.09
Dominant frequencies (1/scan) for MD (1/box) for CD and others		<0.04	<0.1	<0.12	Between 0.11 and 0.12	<0.008	

Table 1 Summary of 2σ's, local variability indices, and the range of dominant frequencies

SUMMARY

The traditional variance partition analysis (VPA) separates a two-dimensional sheet variation in MD, CD and Residual components. VPA does not provide any information regarding the characteristics of the dominant variation contents in MD, CD or Residual. This paper discusses how to use power spectral analysis (PSA) and principal component analysis (PCA) to extract dominant variation patterns from MD, CD and Residual components. The analyses with these two methods yield a lot more insights regarding a 2D variation. The extracted variation patterns and their associated variances, local variability indices and dominant frequencies, are useful for troubleshooting the root causes of sheet variations. Ultimately, this type of analyses will lead to the removal of the detectable variation patterns, which will help papermakers to produce more uniform sheet products.

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